CHAPTER II: DISCONTINUOUS DEFORMATION

II-1 EXPERIMENTAL DEFORMATION

\[ \sigma_c = 0.1 \text{ MPa} \quad \sigma_c = 3.5 \text{ MPa} \quad \sigma_c = 35 \text{ MPa} \quad \sigma_c = 100 \text{ MPa} \]

\( \sigma_c \) is the confining pressure.

\[ \Delta \sigma = \sigma_1 - \sigma_c \]

\[ \varepsilon \]

Differential stress (MPa)

Strain %

Tension fractures
Shear fractures

\[ \sigma_1 \quad \sigma_2 \quad \sigma_3 \]

Dilatancy
Elastic limit
Elastic strain

\[ \text{Strain: } -\Delta V/V_0 \left(10^{-3}\right) \]
**II-2 MOHR ON STRESS**

**II-2-1 THE STRESS ELLIPSOID**

The same reasoning applied to balloon is seawater can be made on small volume of rock in the earth. The state of stress at a point in the Earth can be represented by an ellipsoid: The Stress Ellipsoid. It can be demonstrated that a vector can represent the state of stress on a planar surface. Its magnitude and orientation is dependent on the orientation of the surface.

Some clarification about stress and force:

\[ \sigma = F / S \]

\[ \sigma = \text{Stress} = \lim_{S \to 0} \frac{F}{S} \]

**Mathematical expression of the state of stress at a point**

\[ [\sigma_{ij}] = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix} = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_1 & 0 \\
0 & 0 & \sigma_1
\end{bmatrix} + \begin{bmatrix}
\sigma_{11} - \sigma_1 & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} - \sigma_1 & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma_1
\end{bmatrix} \]

- **Stress tensor**
- **Isotrope component**
- **Deviator**

\[ \sigma_{ij} = \sigma_{ji} \]

- **Invariant**

\[ \sigma_{11} + \sigma_{22} + \sigma_{33} \]

\[ \sigma_i = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \]

- **Isotropic stress**

\[ \sum \text{Principal differential stresses} = 0 \]
The nine components of the stress tensor can easily be derived from knowledge of the stress vector \((\Sigma x, \Sigma y, \Sigma z)\) acting on three perpendicular surfaces.

We all know that the component \((x, y, z)\) of a vector depends on the orientation of the orthogonal reference framework. The same hold true for tensors. It can be shown that by using the principal axes \(s_1, s_2, s_3\) as an orthogonal reference frame then the tensor has only three components which are the principal axis of the stress ellipsoid: \(s_1, s_2, s_3\).

\[
\sigma_1 \quad 0 \quad 0 \\
0 \quad \sigma_2 \quad 0 \\
0 \quad 0 \quad \sigma_3
\]
II-2 MOHR ON STRESS

II-2-3 STRESS ON A PLANAR SURFACE

The problem we want to tackle is relevant to the stability of faults. We have seen that a vector represents the state of stress acting on a planar surface. We can decompose this vector into a normal stress ($s_n$) acting perpendicularly to the fault plane, and a tangential component called the shear stress $\tau$ acting parallel to the fault. The value of these two components is a simple function of the principal axis of the stress ellipsoid and the orientation of the fault...

The sketch on the right represents a planar fault through a solid block. This block is submitted to three couple of force $F_1$, $F_2$, and $F_3$ acting perpendicularly to the block faces. We assume that $F_1 > F_2 = F_3$, and that the fault makes an angle $\alpha$ with $F_1$. What is the normal and shear stress acting on the fault?

First we note that $F_2$ acts on a face perpendicular to the fault. $F_2$ has therefore no contribution on the normal and shear stress acting on the fault plane. We can therefore focus our attention on $F_1$ and $F_3$. Both forces contribute to both the normal and shear stress. Therefore the normal stress corresponds to the sum of the contribution of $F_1$ and $F_3$ to the normal stress ($s_n$), same thing for the shear stress ($F_1$ and $F_3$), note on the top right sketch that $s_n$ and $s_n$ have opposite direction, whereas $F_1$ and $F_3$ have additive contribution.

Decomposition into normal and tangential component...

For $F_1$ we get...

\[ F_{n1} = F_1 \cdot [\sin \alpha] \quad F_{t1} = F_1 \cdot [\cos \alpha] \]

We switch to stresses noting that
\[ S = \frac{\Lambda}{\sin \alpha} \]

\[ \sigma_{n1} = \sigma_1 \cdot \frac{F_1 \cdot [\sin \alpha]^2}{\Lambda} = \sigma_1 \cdot [\sin \alpha]^2 \]

\[ \Rightarrow \sigma_{n1} = \sigma_1 \left[ \frac{1 - \cos 2 \alpha}{2} \right] \]

\[ \tau_1 = \frac{F_1 \cdot [\cos \alpha] \cdot [\sin \alpha]}{\Lambda} = \sigma_1 \cdot \cos \alpha \cdot \sin \alpha \]

\[ \Rightarrow \tau_1 = \sigma_1 \cdot \frac{\sin 2 \alpha}{2} \]

Finally we get the normal stress and the shear stress ...

\[ \sigma_n = \sigma_{n1} + \sigma_{n3} = \sigma_1 \cdot \left[ \frac{1 - \cos 2 \alpha}{2} \right] + \sigma_3 \cdot \left[ \frac{1 + \cos 2 \alpha}{2} \right] \]

\[ \tau = \tau_1 + \tau_3 = \sigma_1 \cdot \frac{\sin 2 \alpha}{2} - \sigma_3 \cdot \frac{\sin 2 \alpha}{2} \]

After minor simplification we get:

\[ \sigma_n = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \cdot \cos 2 \alpha \quad \text{Normal stress} \]

\[ \tau = \frac{\sigma_1 - \sigma_3}{2} \cdot \sin 2 \alpha \quad \text{Shear stress} \]
II-2 MOHR STRESS

II-2-4 THE MOHR’S CIRCLE

We now know how to express the normal stress and the shear stress acting on a fault plane as a function of the principal stresses and the angle $\alpha$ between the fault plane and to the maximum principal stress axis $\sigma_1$.

\[
\sigma_n = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha \quad \text{Normal stress}
\]
\[
\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha \quad \text{Shear stress}
\]

From this we conclude that (1) the maximum shear stress that a fault plane can enjoy is half the difference between $\sigma_1$ and $\sigma_3$. This occurs when the fault is at 45º from $\sigma_1$ ($\alpha=45^\circ$). And the minimum shear stress is zero ($\alpha=0$ or 90º). In contrast the $\sigma_1$ is the maximum normal stress a fault plane can enjoy. This occurs when the fault is at 90º from $\sigma_1$ ($\alpha=90^\circ$). The minimum normal stress is $\sigma_3$ ($\alpha=0^\circ$).

An interesting thing to note about these equation is the two terms: The first represent the centre of a circle, and the second its radius.

This circle is the Mohr circle.

\[
\sigma_3 = \frac{\sigma_1 + \sigma_3}{2}
\]
\[
\frac{\sigma_1 - \sigma_3}{2}
\]

Great but what the...does all this mean? Nothing really except that one can derive a graphical construction to represent the normal and the shear stresses acting on a fault plane. Let’s see if we can figure out this construction...

We start again with our initial problem. A fault embedded in a medium on which a state of stress is applied such that the angle between the fault plane and $\sigma_1$ is $\alpha$. The stress acting on the plane itself is a vector one end of which lies on envelop of the stress ellipsoid the other at the center of the ellipsoid...
II-2 MOHR ON STRESS

II-2-5 THE MOHR’S CIRCLES IN 3D

What if the fault plane does not contain a principal stress axis? Then the total stress acting on the fault lies, on the Mohr diagram, on a surface (in yellow in the sketch below) bounded by three Mohr circles involving two of the three principal stress axes. To determine the exact position of the stress vector one needs to know the angles $\alpha$, $\beta$, and $\gamma$ between the normal to the fault plane and the three principal stress axes $\sigma_1$, $\sigma_2$, and $\sigma_3$ respectively. The construction implies that the total stress vector has one end at the origin of the Mohr diagram whereas its second end lies at the intersection between three circles $P_1$, $P_2$ and $P_3$. The center of those circles lies on the normal stresses axis and their values are: $(\sigma_1+\sigma_2)/2$, $(\sigma_1+\sigma_3)/2$, and $(\sigma_2+\sigma_3)/2$ respectively. Their respective radius are determined by the intersection between:

- the circle $(\sigma_1,\sigma_3)$ and a line extending from $\sigma_1$ at and angle of $\gamma$ from the normal stress axis.
- the circle $(\sigma_1,\sigma_2)$ and a line extending from $\sigma_1$ at and angle of $\beta$ from the normal stress axis.
- the circle $(\sigma_1,\sigma_2)$ and a line extending from $\sigma_3$ at and angle of $\alpha$ from the normal stress axis.

Once the total stress vector is determined one can easily determine its normal and shear component.
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II-2 MOHR ON STRESS

II-2-6 STATE OF STRESS

Uniaxial tension: $\sigma_3 < 0$, $\sigma_1 = \sigma_2 = 0$

Uniaxial compression: $\sigma_3 = \sigma_2 = 0$, $\sigma_1 > 0$

Triaxial tension: $\sigma_1 = \sigma_2 > \sigma_3$

Triaxial compression: $\sigma_3 = \sigma_2 < \sigma_1$
II-2 MOHR ON STRESS
II-2-6 STATE OF STRESS

Simple shear: $\sigma_3 = \sigma_1$, $\sigma_2 = 0$
Shear stress is maximum: $\tau = \sigma_1$
Stress tensor is deviatoric: $\sigma_1 + \sigma_2 + \sigma_3 = 0$

Plane stress: $\sigma_i + \sigma_j = 0$

General stress: $\sigma_1 > \sigma_2 > \sigma_3 > 0$
CHAPTER II : DISCONTINUOUS DEFORMATION

II-3 THE MOHR’S ENVELOPE AND THE COULOMB’S CRITERIA

\[ \tau = C + \mu \sigma_n \]

\( \mu = \tan (\phi) \)

\( C \) : cohesion between crystal at atmospheric pressure
\( \mu \) : coefficient of internal friction (roughness of the shear plane)
\( \phi \) : angle of internal friction

For a diabase: \( C \approx 120 \text{ MPa}, \mu \approx 1 \)

Definition domain of Coulomb’s criteria : \( \tau > C \), brittle deformation

In natural rocks angle of internal friction varies with \( \sigma_n \)
Effect of pore pressure on the shear strength of fault zones. The graph A shows a pore pressure profile. At around 2 km, an impermeable barrier, which prevents fluid escape toward the surface, is responsible for a pore pressure increase. Assuming that a fault zone cuts across this impermeable barrier, the graph B illustrates the dependence of the frictional shear strength of the fault ($\tau$). Variation of shear strength depends on the difference between the pressure normal to the fault plane ($\sigma_n$) and the pore pressure ($P_p$).

$$\tau = C + \mu \sigma_n (\sigma_n - P_p)$$

$$P_{\text{eff}} = \sigma_n - P_p$$

When pore pressure exists, normal stresses $\sigma_n$ are diminished by $P_p$. 
CHAPTER II: DISCONTINUOUS DEFORMATION

II-5 FRACTURING ASSISTED BY PORE PRESSURE

High stress (Mohr circle of large radius) => Rupture by shearing

Low stress (circle of small radius) => Rupture in tension

High pore pressure

\[ \Delta V > 0 \]

\[ P_p = (\rho_2 - \rho_1) \ g \ h \]