COOLING OF A DIKE: ANALYTICAL vs NUMERICAL

The analytical solution that describes the heat transfer through time across of a dike 2w meters wide is:

\[ T(x,t) = T_0 + \left( \frac{T_1 - T_0}{2} \right) \cdot \left( \text{erf}\left( \frac{w - x}{2\sqrt{\kappa \cdot t}} \right) + \text{erf}\left( \frac{w + x}{2\sqrt{\kappa \cdot t}} \right) \right) \]

\( \text{erf}(x) \) is the error function. It was first tabulated in the mid-1800's during the development of probability theory. Its properties are:

- \( \text{erf}(0) = 0 \)
- \( \text{erf}(\infty) = 1 \)
- \( \text{erf}(-x) = -\text{erf}(x) \)

The complementary error function \( \text{erfc}(x) \) is defined as: \( \text{erfc}(x) = 1 - \text{erf}(x) \)

For those of you who have no other way to calculate \( \text{erf} \) function an approximation of the error function is:

- \( 0 \leq x < 0.6: \text{erf}(x) = x \)
- \( 0.6 \leq x \leq 1.2: \text{erf}(x) = 0.84 + \log(x) \)
- \( 1.2 < x \leq 2: \text{erf}(x) = 0.9 + 0.35 \cdot \log(x) \)
- \( x > 2: \text{erf}(x) = 1 \)

with \( \text{erf}(-x) = -\text{erf}(x) \)

- At the time of emplacement the material inside a 20 meter wide dike was at 900°C whereas the surrounding host rock was at 300°C. Using the analytical solution above, calculate the temperature profile across the dike at a time \( t = 1 \) month, 1 year, 10 years after emplacement. (Diffusivity: \( 10^{-6} \text{ m}^2 \cdot \text{s}^{-1} \))

- Build an Ellips model using a 1x2 box, level:4, temperature-independent viscosity (\( \eta_0 = 2500 \)). The dike, 0.2 thick, is centered at \( x = 1 \). Background temperature: 300°C, temperature in the dike: 900°C. Since lengths are divided by 100 the diffusivity must also be scaled. This scaling can be done by noting that the dimension of diffusivity is \( \text{m}^2 \cdot \text{s}^{-1} \). Since the scaling factor for length (m) is \( 10^2 \) that of surface (m²) is \( 10^5 \). The analytical solution tells us that \( T(x, t) \) is depends on \( x/(2\sqrt{\kappa \cdot t}) \). The temperature is not affected by reducing the numerator as long and the denominator is reduced in the same proportion. Compare the analytical and numerical solutions.

Some parameters you want to experiment with:

Fixed time step: You can force Ellips to use a particular time step. You to this by entering:
# Advection Diffusion Parameters:
    fixed_timestep=yourvaluehere

Temperature anomaly, in a rectangular, circular, triangular form: To replace or multiply or increase the temperature of a particular region use the following...

# Field distribution:
    Temp_rect=1
    ...

To see the color of the dike fading as it cools you needs to tun off PPM_coloring_autorange and specify the temperature for the min and max color.

    PPM_coloring_autorange=0,1    # automatically scale colour (default=1)
    PPM_coloring_min=300,0.0     # min value for color scale (default=0.0)
    PPM_coloring_max=900,1.0

If you want to run a model with a small fixed time step (to resolve early temperature gradient) then change the time step to a larger one when the model can afford to (when strong temperature gradient have been smoothed out) do the following: Stop the model, modify the input file (change the fixed_timestep) and call the temperature field of the particular time step you want your experiment to carry on from (temperature field are recorded in #.node_data files). To call a particular #.node_data field as input temperature field use function previous_temperature_files in the initial condition section of the script.

# Initial Condition files:
    ...
    previous_temperature_files="yourtempfile.node_data"