Swash overtopping and sediment overwash on a truncated beach

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Abstract

New experimental laboratory data are presented on swash overtopping and sediment overwash on a truncated beach, approximating the conditions at the crest of a beach berm or inter-tidal ridge-runnel. The experiments provide a measure of the uprush sediment transport rate in the swash zone that is unaffected by the difficulties inherent in deploying instrumentation or sediment trapping techniques at laboratory scale. Overtopping flow volumes are compared with an analytical solution for swash flows as well as a simple numerical model, both of which are restricted to individual swash events. The analytical solution underestimates the overtopping volume by an order of magnitude while the model provides good overall agreement with the data and the reason for this difference is discussed. Modelled flow velocities are input to simple sediment transport formulae appropriate to the swash zone in order to predict the overwash sediment transport rates. Calculations performed with traditional expressions for the wave friction factor tend to underestimate the measured transport. Additional sediment transport calculations using standard total load equations are used to derive an optimum constant wave friction factor of $f_w = 0.024$. This is in good agreement with a broad range of published field and laboratory data. However, the influence of long waves and irregular wave run-up on the overtopping and overwash remains to be assessed. The good agreement between modelled and measured sediment transport rates suggests that the model provides accurate predictions of the uprush sediment transport rates in the swash zone, which has application in predicting the growth and height of beach berms.

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1. Introduction

Many natural beaches exhibit a wave-built berm landward of the intersection of the mean high water level and the beach profile (Komar, 1998; Masselink and Hughes, 2003). The berm frequently acts as a barrier to swash action, protecting lower lying topography in the backshore region, and also acts as a sediment supply to the foreshore and surf zone during periods of beach erosion when the berm is cut back or reduced in height. Following such events, the beach berm typically recovers, growing in height, seaward extent or both. Swash zone sediment transport strong-
ly determines this recovery process (Elfrink and Baldock, 2002). Beach nourishment may also involve the construction of beach berms, or alternatively, the placement of sand offshore with the aim of wave action moving it onshore to form a natural berm (Dean, 2002). In addition, the subsequent aeolian transport of sediment deposited on the upper beach face is critical to foredune growth and stability (Aagaard et al., 2004). The rate of berm growth is therefore important for understanding and predicting the beach recovery process, and is essential for longer term sustainable management of the coastal zone.

While some berm growth may be due to aeolian sand transport, growth in berm height is usually wave driven, which requires that wave run-up must overtop the berm, depositing sediment beyond the existing crest. This process is often linked to the spring–neap tidal cycle, with small berms formed on the foreshore at lower tidal ranges pushed landward and upwards with the rise to spring tides (Hughes and Turner, 1999). Sediment transport above the local mean water level is due to swash; i.e. the balance between uprush and backwash sediment transport on the beach face. Uprush sediment transport and overwash at the position of the existing berm crest is therefore the key to determining berm growth (Baldock et al., 2004b).

The overtopping flow itself may also be important in determining flooding of the crest and landward face of berms, dunes or gravel barriers (Orford et al., 1991; Kobayashi and Tega, 1996), and infilling of the runnel behind intertidal ridges (Aagaard et al., in press). However, reliable sediment transport predictions in the swash are beyond the state of the art (see Elfrink and Baldock (2002) for a recent review), with much uncertainty surrounding key parameters such as flow velocities and appropriate friction factors (e.g. Cox et al., 2000; Puleo and Holland, 2001; Raubenheimer et al., 2004; Hughes and Baldock, 2004; Conley and Griffin, 2004).

Laboratory measurements of swash hydrodynamics and sediment transport are difficult to obtain, due to the small flow depths and thin layers of moving sediment. In the field, flow depths are less restrictive, but conditions are much more variable and it is difficult to obtain simultaneously flow velocities, flow depths, sediment transport rates and the incident forcing conditions (see Butt and Russell, 2000 for a general review). The difficulties in predicting swash sediment transport are strikingly illustrated by the problem of determining appropriate friction factors. For example, based on vertical flow structures and turbulent dissipation, Raubenheimer et al. (2004) inferred swash friction factors in the range \( f = 0.02 – 0.06 \), whereas direct measurements of bed shear by Conley and Griffin (2004) gave inferred friction factors an order of magnitude smaller.

Here, inspired by the swash overtopping solution of Peregrine and Williams (2001), we present novel uprush sediment transport data obtained by truncating the beach at different elevations above the SWL and allowing the flow and sediment to overtop the beach. The sediment transport or overwash is simple to obtain on the truncated beach and represents a simplification of the conditions at the crest of a beach berm. The results, however, indicate that the analytical solution of Peregrine and Williams (2001) underestimates the overtopping flow by an order of magnitude. The reason for this is illustrated and it results from an approximation for the flow depth introduced by Shen and Meyer (1963). Consequently, further comparisons are made with a numerical model based on the work of Baldock and Holmes (1997), recently extended and further verified for natural swash by Hughes and Baldock (2004). The model results compare well with the overtopping data and a simple regression formula is proposed to improve the accuracy of the Peregrine and Williams (2001) solution. Finally, sediment transport rates estimated from standard formulae are calculated using flow velocities predicted by the model, and they are generally in good agreement with the measured overwash data. Data are obtained from cross-shore locations covering approximately the three-quarters of the swash zone up to the run-up limit.

In Section 2 we briefly review the solution of Peregrine and Williams (2001) and the basis for the numerical model. Section 3 provides a description of the experimental arrangement and wave conditions used in the study. Section 4 presents and discusses the experimental data, and provides model–data comparisons for both swash overtopping and overwash sediment transport. Final conclusions follow in Section 5.
2. Theory and modelling

2.1. Background theory

Peregrine and Williams (2001), hereafter PW01, presented an elegant analytical solution for inviscid swash overtopping a beach cut-off at some elevation below the usual maximum run-up point. The solution is an extension of the swash solution given by Shen and Meyer (1963), hereafter SM63, applicable for broken wave bores approaching the shoreline. Swash due to non-breaking standing waves is excluded. While standing waves frequently contribute to swash overtopping due to irregular waves (Williams and Peregrine, 2002), any additional flow arising from these waves is not accounted for by SM63 or PW01. Similarly, full numerical calculations (Williams and Peregrine, 2002) here we consider only monochromatic breaking wave conditions so standing wave swash is excluded and the analytical swash solutions are assumed to be applicable. PW01 recast the SM63 swash solution in dimensionless form and eliminated beach slope from the equations, such that the maximum runup elevation above the initial shoreline position is $2A$ and the swash period becomes $T_s=4$.

The equation for the non-dimensional shoreline motion, $x_s(t)$ is parabolic and becomes

$$x_s(t) = 2t - \frac{1}{2} t^2$$

Maximum run-up occurs at $t=2$, and the maximum swash excursion along the beach is equal to $x_s=x_{\text{max}}=2$. Thus the midpoint of the swash corresponds to $x=1$. The non-dimensional water depth, $h$, close to the shoreline is given by (SM63, Meyer and Taylor, 1972)

$$h(x,t) = \frac{1}{9t^2} (x_s(t) - x)^2$$

Using Eq. (1), Eq. (2) is written as

$$h(x,t) = \frac{1}{36t^2} (4t - t^2 - 2x)^2$$

which is similar for all swash events. The dimensional depth, $h^*$, is $hA / \cos(\beta)$, where $\beta$ is the beach slope, and so $h^* = hA$ for most practical beach slopes. In dimensional terms and for inviscid swash, the theoretical run-up height, $R$, for a bore of height $H_b$ is simply $R = 2H_b$, so $A = H_b$ and the swash period $T_s^* = 4\sqrt{gH_b}/g\beta$. At the mid-swash position at the time of maximum run-up Eq. (3) gives $h = 1/36$ or $h^* = H_b/36$; a very thin swash.

PW01 extended this solution to derive an explicit expression for the flow velocity, which is again similar for all swash events

$$u(x,t) = \frac{2}{3t} (t - t^2 + x)$$

The non-dimensional flow rate per unit width at any point, $q = uh$, is calculated readily from Eqs. (3) and (4). PW01 subsequently considered the swash flow arising if the beach is truncated at a point $x=E$ below the usual maximum run-up limit, such that the flow falls freely over the edge. Up to a time $t = \sqrt{2E}$, the flow is supercritical and unaffected by the edge. After this time the edge acts as a hydraulic control and the flow remains critical at that point, thus controlling the flow further downslope. PW01 provide a solution for $h(x,t)$ and $u(x,t)$ for these conditions until the overtopping flow at $x=E$ ceases at $t=2$. The total overtopping flow volume, $V(E)$, is then obtained over the two time intervals and is a simple function of $E$ only

$$V(E) = \frac{1}{27} \left( 4 - 12E + 8E\sqrt{2E} - 3E^2 \right)$$

For reference, the dimensional form of Eq. (5) is $V^*(E)=2A^2V(E)/\sin (2\beta)$, where $\beta$ is the beach slope. It is important to note that Eqs. (2) and (3) are only expected to be accurate close to the moving shoreline and that Eqs. (2)-(4) are not useful close to the singularity at $(x,t)=0$ (PW01).

2.2. Modelling

Given the limitations of Eq. (2) Baldock and Holmes (1997) proposed a similarity solution for the swash given by:

$$h^*(x,t) = H_b \left( \frac{x_s-x}{x_s} \right)^C \left( \frac{T_s-t}{T_s} \right)^D$$

where $x_s$, $x$, $t$ and $T_s$ may be dimensional or dimensionless as required. As with PW01, $x=t=0$ repre-
sents the start of bore collapse. \( C \) and \( D \) are empirical coefficients that are expected to vary with beach slope and wave conditions and are in the range \( C = 0.5 - 0.75 \) and \( D = 1 - 2 \). Eq. (6) is formulated for use with parametric sediment transport models or broad-scale morphological models where full numerical time stepping hydrodynamic models are computationally too expensive. As with Eq. (3), Eq. (6) gives a similar swash profile for all events. For \( C = 0.75 \) and \( D = 2 \), appropriate for \( \beta = 0.1 \) (Baldock and Holmes, 1997), Eq. (6) gives \( h^* = 0.15H_b \) (\( x = 1 \)) at the time of maximum run-up, a factor 5 times greater than that predicted by Eq. (3). This factor increases with lower values of \( C \) and \( D \), which are appropriate for milder beach slopes (Hughes and Baldock, 2004). The flow velocity \( u^*(x,t) \) may be obtained by integrating Eq. (6) between \( x \) and \( x_s \) at two different time steps and then using a simple finite difference approach to determine the depth averaged flow velocity

\[
u^*(x,t) = \frac{V_s(t + \Delta t) - V_s(t - \Delta t)}{2h_s(x,t)\Delta t}
\]

(7)

where \( V_s \) is the volume of water shoreward of \( x \). Baldock and Holmes (1997) showed good agreement between Eq. (7) and laboratory data, and Hughes and Baldock (2004) show that Eqs. (6) and (7) also compare well with field observations of swash kinematics. For comparison with SM63 and PW01, Eq. (6) is non-dimensionalised according to \( h = h^* \cos \beta / H_b \) and (7) according to \( u = u^* / \sqrt{gH_b} \). At the run-up tip, \( x = x_s, u^* \) is equal to the shoreline velocity, which at the initiation of the swash is equal to \( 2\sqrt{gH_b} \). Thus \( h_{\text{max}} = 1 \) and \( u_{\text{max}} = 2 \). Note that Eqs. (3) and (4) only apply to inviscid swash, whereas Eq. (6) was derived on the basis of viscid laboratory data. For consistency, therefore, when comparing numerical calculations with the SM63 and PW01 solutions the model calculations were performed for an inviscid swash solution for the shoreline motion, i.e. using Eq. (1).

To illustrate, Fig. 1a compares the non-dimensional swash depth from Eq. (6) with laboratory data from a plane beach (\( \beta = 1/10 \)), with good agreement. Fig. 1b compares Eq. (6) with Eq. (3) at the mid-swash position, \( x = 1 \). Clearly (3) gives much smaller flow depths, and they appear unrealistically small. In
dimensional terms for example, with a bore of height 0.5 m at the shoreline, representative of relatively energetic field conditions, Eq. (3) predicts the maximum depth at the mid-swash position to be approximately 0.02 m. In contrast, field data suggest typical maximum mid-swash depths of order 0.05–0.2 m (e.g. Hughes, 1992; Puleo and Holland, 2001; Raubenheimer, 2002; Hughes and Baldock, 2004). For the same initial bore height, Eq. (6) gives a maximum mid-swash depth of 0.12 m, more consistent with the field observations. Similarly, full numerical calculations give dimensionless values for $h_c \approx 0.2–0.4$ at the mid-swash position (Masselink and Li, 2001; Duy et al., 2002), in agreement with Eq. (6) on Fig. 1b. Fig. 2a and b compare the non-dimensional water surface elevation, $z$, across the swash zone predicted by Eqs. (3) and (6), where $z=0$ and $z=2$ correspond to the beach elevations at the start of the swash event and at the maximum run-up elevation, respectively. In comparison with Eq. (3), Eq. (6) gives a much greater volume of water in the uprush, leading to significantly greater predicted overtopping volumes (see below).

The analytical solution for the swash flow field given by PW01, Eq. (4), can provide estimates for a number of important characteristics of swash hydrodynamics (see Baldock and Hughes, submitted for publication). For example, from Eq. (4) the local temporal acceleration ($\partial u/\partial t$) at any point is always negative, i.e. offshore. This is consistent with recent laboratory and field data presented by Petti and Longo (2001) and Hughes and Baldock (2004). The cross-shore variation in velocity moments (e.g. $<u^3>$) are also estimated readily from Eq. (4). Furthermore, in contrast to the depths predicted by Eq. (3), the flow velocity from Eq. (4) compares well with that from the numerical model Eq. (7), except close to the singularity at $x=0$. This is illustrated in Fig. 3a–c, where shoreward of $x=1$ the differences between Eqs. (4) and (7) are relatively minor. Hence, any differences between overtopping flows predicted by Eqs. (3) and (4) and by Eqs. (6) and (7) are likely to be due to differences in predicted flow depths rather than flow velocities. This point is considered further below with reference to the new laboratory data.

3. Experiments

The experiments were carried out in the Coastal Engineering wave flume in the Civil Engineering Department at the University of Queensland. The flume is 20 m long and 0.9 m wide, and was used with water depths ranging from 0.3–0.45 m (Fig. 4). A plane beach, gradient $\beta=0.1$, was installed 7 m from the wavemaker and truncated at an elevation of 0.5 m. The truncated edge was sharp and cut perpendicular to the beach face. Overtopping flow volumes were collected in a measuring tank positioned below the truncated beach, with the volumetric flow rate determined from the change in depth in the tank. Offshore wave conditions 4 m seaward from the beach toe were
measured with an electronic wave gauge. The reflection coefficient for the wave conditions considered ranged from 5%–12% in the absence of overtopping. For selected cases the maximum flow depth at the truncation point was also measured with a sharp-tipped surface elevation dial gauge. Run-up, overtopping and overwash measurements were obtained for offshore wave heights ranging between 0.04 and 0.18 m, with periods varying from 1.5 to 3.0 s. A larger number of tests was carried out for the overtopping experiments, with a second data set collected after the overwash experiments to verify the discrepancy between the measured volumes and those predicted by Eq. (5). The duration of each test varied with wave period and overtopping volume to ensure accurate measurement of the total overtopping volume, typically 20–100 waves.

A non-dimensional measure of the truncation point, \( E \), varies between 0 (corresponding to the SWL or wave run-down position) and 2 (at the expected usual run-up limit, \( R \)). For each wave condition, \( R \) was determined by running the same wave conditions on a non-truncated beach (using a lower water level), with the data comparing well with Hunt’s (1959) formula \( R = \sqrt{(H_o L_o) \tan \beta} \), where \( H_o \) and \( L_o \) are the deep water wave height and wave length respectively). Since either \( R \) or \( H_b \) uniquely determines the swash solution, the scaling may be based on either the bore height at the initial shoreline or the offshore wave height. In the latter case, which is particularly convenient for practical applications, \( R \) may be estimated from Hunt’s formula or other parameterisations of run-up (e.g. Holman and Sallenger, 1985). This overcomes the need to apply a surf zone transformation model. Theoretically, swash interaction occurs if \( R \) exceeds the saturated run-up height, \( R_s \), given by Baldock and Holmes (1999):

\[
R_s \leq \frac{g h^2}{8f^2}
\]  

where \( f \) is the frequency of the incident bores or swash period. However, the overtopping reduces the swash period (PW01) and reduces swash interaction, which did not occur for the majority of the wave conditions used.

Overtopping flow volumes were first collected with a smooth fixed beach, i.e. in the absence of sand. A 50 mm thick sand layer (0.3 mm median grain size) was then added parallel to the underlying beach and the wave conditions repeated for a selected number of tests, with the bed levelled between each run. Sand avalanching at the truncation point was prevented with a 50 mm high “toe board”. Since the net transport at the truncation point was always landward, the beach remained plane and level with the “toe board” at this point, and no scour effects were observed. The “toe board” never protruded above the sand surface hindering the overwash, nor does the sand level exceed that of the “toe board” by more than a millimeter or so. Hence slumping or avalanch-
ing does not occur. Beach profile change was measured across the surf and swash zone using a simple drop stick. Small changes in the initially plane beach profile occurred both in the surf and swash zones. Variations in sand surface elevation in the swash zone typically ranged from 0 to 10 mm, with the formation of a beach step usually occurring in the surf zone. The degree of profile change depended on the run time, which varied between 1–5 min, depending on the overwash load. The profile change on the truncated beach was similar to that observed for the same wave conditions in the absence of sediment overwash. Hence, although the position of the truncation point does influence the profile change, over short durations the uprush transport at the truncation point is expected to be similar to that on a non-truncated beach.

Overtopping sediment was trapped by a fine-gauge mesh collecting tray positioned below the truncated edge. It was subsequently dried and weighed to derive the total uprush transport per wave. For both sets of experiments the truncation point was varied by changing the water level in the flume, with $E$ ranging between 0.5 and 2. For $E < 0.5$, the beach would be truncated within the lower quarter of the swash zone and overtopping would occur almost before bore collapse was complete (see Fig. 2). Such conditions seem unlikely to occur during the process of berm building on natural beaches.

4. Results and applications

4.1. Overtopping flows

PW01 show that the flow at the truncation point is not expected to be influenced by the edge until critical conditions are reached. This occurs when the flow velocity equals the shallow water wave speed ($c^* = \sqrt{gh^*}$, $c = \sqrt{h}$) at time $t = \sqrt{2E}$. PW01 modify Eqs. (3) and (4) to account for the edge condition, but this is presently excluded from the model calculations. This omission is considered acceptable for three reasons. First, PW01 show that the edge only has a minor effect on the overtopping flow for $E > 0.5$. Second, in comparison to differences between the analytical prediction from Eq. (3) and model predictions from Eq. (6), any modification to the model flow depth will lead to negligible changes in overtopping volume, since the flow velocity reduces rapidly to zero. Finally, close inspection of the overtopping flow suggests the depth does not reduce to zero at the instant the overtopping flow ceases, as predicted by PW01. The latter two points are considered further below.

Fig. 5 compares the maximum measured flow depth at the truncation point with the theoretical prediction from Eqs. (3) and (6). Consistent with Fig. 1, the measured data are much greater than those predicted by the analytical solution and in good agreement with the similarity solution proposed by Baldock and Holmes (1997). For $E \approx 0.5$, however, Eq. (6) overpredicts the maximum depth at the truncation point, which might be expected if the edge is acting as a hydraulic control. Fig. 6a shows the predicted flow depth and velocity at the truncation position when $E=1$. In comparison with Figs. 1b and 3b, the PW01 solution is modified after time $t = \sqrt{2E}$ such that the flow velocity reduces to zero at time $t=2$, rather than at $t \approx 1.62$. The numerical solution is unaltered, but truncated at the instant the flow velocity reaches zero at $t \approx 1.6$. The numerical calculations predict that critical conditions are reached at $E=1$ at $t \approx 1.2$ s, indicated on Fig. 6b when $u=c$. After this time, the flow velocity is small in comparison to the earlier part of the uprush and thus the flow discharge, $q=uh$, that occurs for $1.2 \leq t \leq 1.6$ is small compared to the total. Consequently, neglecting the edge effects is likely to introduce only minor errors.

Fig. 7a illustrates this further and shows the temporal variation in the predicted overtopping flow, $Q_o(t)$, again for $E=1$. The model calculations based on (6) and (7) give an order of magnitude greater overtopping flow rates than the PW01 solution.
Eq. (5). Consistent with the analysis in Section 2 and Fig. 5 above, this is due to the greater depths predicted by Eq. (6) in comparison to Eq. (3). Fig. 7b compares the total non-dimensional overtopping volume per wave, $V(E)$, from Eq. (5) with the numerical calculations and the measured data. Despite some scatter, the data are in much closer agreement with the numerical calculations than the PW01 solution. For $1 < E < 2$, this is shown more clearly on a logarithmic plot (Fig. 7c).

The advantage of the PW01 solution is the simplicity of the result, Eq. (5). A scaling ratio has therefore been derived from a regression analysis of the data, the PW01 solution and the numerical calculations. The resulting curve provides an upper bound for the data set and is in close agreement with the full model calculations. The regression model is given by

$$V'(E) = 7.5V(E)(1 + E^3)$$

and also shown on Fig. 7c. On the basis of the present data Eq. (9) is more accurate than Eq. (5) for simple estimates of the overtopping volume for any given truncation point. Finally, video analysis of the overtopping suggests that the depth is small but non-zero when the overtopping ceases. This implies the discharge ceases due to flow reversal occurring prior to the time of maximum uprush, a well known feature of the swash (Larson and Sunamura, 1993; Hughes et al., 1997; Baldock and Holmes, 1997). Therefore, truncating the model calculations with a finite depth at the overtopping point (Fig. 6b) appears to be a reasonable approximation and will only introduce minor errors. For example, in comparison with the regression anal-

![Fig. 6. a. Non-dimensional swash depth and velocity at the overtopping position, $E=1$. —— $u$, PW01, - - - - model, — — $h$, PW01, - - - - $h$, model. b. Non-dimensional swash depth, velocity and shallow water wave speed at overtopping position, $E=1$. - - - - $u$, model, - - - - $h$, model.](image1)

![Fig. 7. a. Variation of non-dimensional overtopping volume with time, $E=1$. —— PW01, - - - - model. b. Variation of total non-dimensional overtopping volume with truncation point $E$. □□ measured, —— PW01, - - - - model. c. Variation of total non-dimensional overtopping volume with truncation point $E$. □□ measured, —— PW01, - - - - model, — — regression curve.](image2)
ysis, the numerical calculations do appear to under-
estimate the overtopping flow to some extent, which is
consistent with truncating the flow at the instant of
flow reversal in the model (Fig. 6a).

4.2. Sediment overwash

Fig. 8 shows that the measured sediment over-
wash, $Q_{bm}$, varies with $E$ in a similar manner to the
measured non-dimensional overtopping flow, $V(E)$,
as expected at the outset of the experiments. How-
ever, the sediment overwash is not a simple function
of $E$, as is the case for the overtopping flow, for
two reasons. Firstly, the bed load transport is largely
independent of the flow depth; secondly, suspended
sediment concentrations vary with time during the
overtopping. This increases the scatter in the data.
Nevertheless, the best fit lines through the data
confirm that both $Q_{bm}$ and $V(E)$ vary similarly
with $E$. The measured sediment overwash is equi-
valent to the sediment transport past the truncation
point during swash uprush. Since the truncation
point is varied, then the experiments provide a
direct measure of the total uprush transport at dif-
ferent cross-shore locations. This approach provides
a simple measure of the transport rate that is not
affected by instrumentation or sediment trapping
techniques.

Theoretical total load sediment transport rates past
the truncation point were calculated for each individ-
ual wave condition using the velocity field predicted
by the numerical model. Two sediment transport
models were considered; a standard sheet flow Shields
model for sediment transport (Nielsen, 1992) and a
modified form accounting for potential flow acceler-
ation in the swash (Nielsen, 2002). The standard sheet
flow transport rate $Q_b$ is:

$$Q_b = \Phi d \sqrt{(s - 1)gd}$$  \hspace{1cm} (10)

with

$$\Phi = C(\theta - 0.05) \sqrt{\nu}$$  \hspace{1cm} (11)

and

$$\theta = \frac{1/2 f_w u|u|}{(s - 1)gd}$$  \hspace{1cm} (12)

where $d$ is the median grain diameter, $s$ is the specific
gravity of the sediment, $u$ is the free stream velocity,
$f_w$ is the wave friction factor and $C$ is an empirical
transport coefficient. Previous estimates of $f_w$ for the
swash zone range from 0.001 to 0.06 (Cox et al.,
2000; Raubenheimer et al., 2004; Conley and Griffin,
2004), potentially leading to large differences in the
transport rates given by Eqs. (10)–(12). Similarly,
estimates of the multiplier $C$, typically 12 for steady
flow data (Nielsen, 1992), range from 8–20 in the
swash (Masselink and Hughes, 1998). A standard
expression for $f_w$ is (Nielsen, 1992, 2002):

$$f_w = \exp \left[ 5.5 \left( \frac{2d_{50}}{A_{rms}} \right)^{0.2} - 6.3 \right]$$  \hspace{1cm} (13)

where $A_{rms}$ is based on the free stream velocity var-
ance and the swash frequency $\omega = 2\pi/T_s$.

$$A_{rms} = \frac{\sqrt{\pi} \sqrt{\text{var}\{u(t)\}}}{\omega}$$  \hspace{1cm} (14)

$A_{rms}$ and $f_w$ therefore vary across the swash zone; for
the present data $f_w$ varied between 0.008 at the SWL
and 0.016 at the run-up limit.

Nielsen (2002) modified Eq. (12) for the case
where a significant positive local temporal flow ac-
celeration or landward directed pressure gradient
and Puleo et al. (2003) proposed that these conditions
may occur during uprush and to a lesser extent
during the backwash. However, neither the solution
of PW01 nor the numerical calculations predict
positive flow accelerations. Similarly, full numerical
calculations based on the shallow water wave equa-
tions for non-interacting swash indicate only nega-
tive flow accelerations (e.g. Hibberd and Peregrine,
Positive flow accelerations are also not observed in the laboratory data presented by Petti and Longo (2001) or the field data presented by Hughes and Baldock (2004). Baldock and Hughes (2005) present further field data on this point. Nevertheless, it is useful to consider the change in transport rate predicted by the Nielsen (2002) approach.

The modified form of Eq. (12) is

\[
\theta_a = \frac{1}{2f_w} \left( \frac{A_{ms} \omega_p}{A^{\alpha}} \right)^{2-n} \left( \cos \phi(t) + \sin \phi \frac{1}{\omega_p} \frac{\partial u}{\partial t} \right)^{2-n} \times \text{sign} \left( \cos \phi(t) + \sin \phi \frac{1}{\omega_p} \frac{\partial u}{\partial t} \right)
\]

noting corrections to the expression given in the original paper (Nielsen, 2004, pers. comm.).

Nielsen (2002) found good agreement with the sediment transport data of Masselink and Hughes (1998) using a phase shift \(\phi = 40^\circ\) and \(n = 0.1\). For the case where \(\partial u/\partial t\) is always negative, however, the magnitude of \(\theta_a\) is significantly reduced compared to Eq. (12), rather than increased. This is illustrated in Fig. 9, which compares Eqs. (12) and (15) at the mid-swash position for the case of no overtopping \((x=1)\). With the inclusion of the threshold criterion, \(\theta_c = 0.05\), the uprush transport rates predicted by Eqs. (10), (11) and (15) become up to two orders of magnitude smaller than those predicted using Eq. (12). For \(\partial u/\partial t \leq 0\) Eq. (15) also predicts a negative transport rate for small, but positive \(u(t)\), which cannot be physically correct for the flow at the edge. Hence Eq. (15) should be used with caution over much of the swash zone given its apparently erroneous predictions for negative flow acceleration and the uncertainties relating to the importance of positive flow acceleration. However, Eq. (15) may well be appropriate at the inner surf zone/swash zone boundary during swash interaction when an uprush bore propagates over the backwash from the preceding wave prior to reaching the exposed beach face. Baldock and Holmes (1997) illustrate the rapid change in flow direction that accompanies such an event, and which implies a corresponding shoreward local acceleration.

Sediment transport calculations based on Eqs. (10)–(14) were carried in two slightly different ways. Firstly, Eqs. (13) and (14) were allowed to vary across the swash zone, depending on the local velocity field at any fixed position. Eq. (11) was then used with \(C = 20\), which is the value providing the best fit to the data of Masselink and Hughes (1998). Secondly, the calculations were repeated with \(C = 12\), the standard steady flow coefficient based on extensive data sets (see Nielsen, 1992), but with \(f_w\) fixed. The bed slope was also accounted for by modifying the Shields parameter according to Fredsøe and Deigaard (1992)

\[
\theta_s = \theta \left( 1 - \frac{\tan \beta}{\tan \phi} \right) \cos \beta
\]

where \(\phi\) is the friction angle (\(\tan \phi = 0.63\)). The value of \(f_w\) was then chosen to provide the best match between the predicted transport and the present overwash data, giving \(f_w = 0.024\). This value lies at the lower end of the range \((0.02 < f_w < 0.05)\) of local swash friction factors inferred by Cox et al. (2000), Petti and Longo (2001) and Raubenheimer et al. (2004). Fig. 10 shows a comparison of the model...
predictions for the dimensional overtopping volume, $Q_w$, and dimensional sediment overwash, $Q_b$ for one set of incident wave conditions. The model predicts a similar correlation between $Q_w$ and $Q_b$ to that shown by the data (Fig. 8). The mean sediment concentration in the overwash is predicted to be 0.032, which is similar to that observed by Kobayashi and Tega (1996) during dune overtopping experiments. Those concentrations were measured at the base of the landward side of the dune, however, and consequently they may be applicable to the sediment transport down the rear of the dune rather than the overwash, or a combination of both.

Fig. 11a compares the measured and predicted transport rates with variable $f_w$ and $C=20$, while Fig. 11b shows the same data versus the model predictions with $f_w=0.024$, $C=12$ and accounting for the bed slope. In the first instance the model underpredicts the data, but shows good agreement when the bed slope is included and with $f_w$ optimised. For $E=1$, the value $f_w=0.024$ is approximately twice that obtained from the standard expression for $f_w$ obtained from Eq. (13), which gives a local swash friction factor considerably lower than those in the literature noted above. Since Eq. (13) was used by Masselink and Hughes (1998) in obtaining $C=20$ in Eq. (11), this may explain their increase in $C$ above the usual value of $C=12$. Thus $C$ is dependent on the choice of $f_w$ when calibrating or applying sediment transport models. The optimum value of $f_w=0.024$ required to match the data is close to the mean value of $f_w=0.03$ observed by Raubenheimer et al. (2004). Both values, however, are an order of magnitude higher than that inferred by Conley and Griffin (2004) and this discrepancy remains unresolved. The transport models above are most appropriate for bed load transport, although they have also been commonly applied to describe the total load (e.g. Masselink and Hughes, 1998; Nielsen, 2002). It should be noted that the grain size in this study roughly corresponds to gravel at prototype scale, where bed load dominates. Hence, some caution should be used if extrapolating the data or friction factors to natural sand beaches where suspended load may well be significant. Nevertheless, the optimum friction factor appears consistent with field data. Further work in a later paper will extend the present approach using data for sediment overwashing the berm crest on a natural beach.

Finally, it should be noted that during the review of this paper, Pritchard and Hogg (2005) presented an extension of the Peregrine and Williams solution to include suspended sediment transport. The equations generally require numerical solutions which preclude a simple comparison with the data here, but future work will attempt to compare their graphical results with the present data. However, the instantaneous sediment fluxes derived by Pritchard and Hogg (2005) are linearly dependent on the flow depth. Consequently, the apparent underestimation of the flow depths by the analytical solution may lead to an underestimate of the instantaneous suspended sediment flux, particularly during the uprush.

5. Conclusions

New laboratory data have been presented on swash overtopping and sediment transport on a truncated
beach, conditions which approximate the crest of a beach berm or inter-tidal ridge-runnel. The overtopping data are compared to the analytical solution of Peregrine and Williams (2001) and a simple numerical model (Baldock and Holmes, 1997; Hughes and Baldock, 2004). The analytical solution underestimates the overtopping flow by an order of magnitude, largely as a result of the very small flow depths given by the swash solution. In contrast, the model predictions are in good agreement with the data. The flow velocity given by the analytical solution is, however, similar to that predicted by the numerical model.

By truncating the beach at different elevations above the SWL the experiments provided a measure of the uprush sediment transport rate at different cross-shore locations in the swash zone. In addition, the measurements were unaffected by the difficulties inherent in deploying instrumentation or sediment trapping techniques at laboratory scale. The flow velocity derived from the numerical model is used in conjunction with simple total load transport formulae to predict sediment overwash or sediment transport rates during wave uprush. A standard expression for the wave friction factor gives $0.008 < f_w < 0.016$ and leads to an under prediction of the measured data when combined with sediment transport coefficients that appear appropriate for the swash zone. Subsequent sediment transport predictions using standard transport coefficients for steady flows are used to derive an optimum constant wave friction factor of $f_w = 0.024$. This is in good agreement with a broad range of published field and laboratory data. Finally, the good agreement between modelled and measured sediment transport rates suggests the model provides accurate predictions of the uprush sediment transport rates in the swash zone and thus may be useful in modelling the growth and height of beach berms or the infilling of runnels.

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References


