Settling velocity of sediments at high concentrations

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Received 17 June 2003; received in revised form 10 December 2003; accepted 19 December 2003

Abstract

New data on the settling velocity of artificial sediments and natural sands at high concentrations are presented. The data are compared with a widely used semiempirical Richardson and Zaki equation (Trans. Inst. Chem. Eng. 32 (1954) 35), which gives an accurate measure of the reduction in velocity as a function of concentration and an experimentally determined empirical power $n$. Here, a simple method of determining $n$ is presented using standard equations for the clear water settling velocity and the seepage flow within fixed sediment beds. The resulting values for $n$ are compared against values derived from new and existing laboratory data for beach and filter sands. For sands, the appropriate values of $n$ are found to differ significantly from those suggested by Richardson and Zaki for spheres, and are typically larger, corresponding to a greater reduction in settling velocity at high concentrations. For fine and medium sands at concentrations of order 0.4, the hindered settling velocity reduces to about 70\% of that expected using values of $n$ derived for spheres. At concentrations of order 0.15, the hindered settling velocity reduces to less than half of the settling velocity in clear water. These reduced settling velocities have important implications for sediment transport modelling close to, and within, sheet flow layers and in the swash zone.

Keywords: Fall velocity; Hindered settling; Sediment concentration; Suspended sediment transport; Sheet flow; Swash zone sediment transport

1. Introduction

An accurate assessment of the settling velocity of sediment particles is fundamental to the modelling of sediment suspension, mixing processes and sediment transport in the coastal zone. For single particles, or dilute suspensions, the settling velocity can be accurately predicted by equating gravity and drag forces using an appropriate drag coefficient for spheres (e.g. Nielsen, 1992) or sand (e.g. Fredsøe and Deigaard, 1992). Alternatively, a number of empirical expressions for the fall velocity are available (Gibbs et al., 1971; Jiminez and Madsen, 2003). However, in high energy flow conditions and close to the sediment bed, suspended sediment concentrations become significant, approaching the concentration of the immobile bed itself, $c_{\text{max}}$. For example, in sheet flow layers and plug flow, concentrations range from $c_{\text{max}}$ to about $0.1c_{\text{max}}$ (Zala Flores and Sleath, 1998; O’Donoghue and Wright, 2001, Dohmen-Janssen and Hanes, 2002). At these concentrations, the particle settling velocity may reduce to 10\% or less of the clear water settling velocity (hindered settling), leading to reduced (less negative) concentration gradients over
those otherwise expected, i.e. a more uniform distribution of sediment in the vertical. In addition, Sleath (1999) notes that the sediment settling velocity at these high concentrations is an important factor controlling the compaction of the sediment bed and the subsequent sediment mobility.

Recent advances in sediment transport modelling are beginning to consider hindered settling effects (e.g. Li and Davies, 2001), and the next generation of sediment transport models will need to advance this further. However, the need for an improvement upon Richardson and Zaki (1954) was pointed out by Nielsen et al. (2002) since that and most later work (e.g. Khan and Richardson, 1989) focused on spherical particles. Therefore, a simple but accurate method to estimate the settling velocity of natural beach sediments at high concentrations would appear beneficial. This paper discusses such a method, which in fact had been considered previously but does not appear at all well known in the literature, especially with regard to coastal sediment transport where it is particularly relevant. The following section outlines previous work and the basis for estimating the settling velocity at high concentrations. A description of some recent experiments used to verify the equations developed below is given in Section 3. Section 4 presents and discusses the experimental data, with final conclusions presented in Section 5.

2. Background

2.1. Previous work

A large number of experiments have demonstrated that particle settling velocities are lower at higher concentrations by a factor usually given by the widely used semiempirical Richardson and Zaki (1954) equation:

$$\frac{w_s}{w_t} = \varepsilon^n = (1 - c)^n$$

(1)

where $w_s$ is the settling velocity at voidage $\varepsilon$ (porosity in the soil mechanics literature), $c$ is volumetric concentration and $w_t$ is the terminal settling velocity in an infinite fluid. $n$ is an empirically determined exponent dependent on the particle Reynolds number, $R_t$, at $w_t$, and is constant for a particular particle. Eq. (1) was found to hold over the full range of possible voidage above incipient fluidisation and $n$ was determined experimentally to lie in the range 4.65–2.4 for increasing $R_t$. Experiments since and further correlations of the data have not significantly improved the predictive capability of Eq. (1) (e.g. Rowe, 1987; Khan and Richardson, 1989), and an extensive review of suggested empirical expressions for $n$ from the Chemical Engineering literature is given by Di Felice (1995). It should be noted that in Eq. (1), the empirical values describing the variation in $n$ with $R_t$ were derived on the basis of fluidisation and sedimentation experiments with spheres. To our knowledge, no equivalent measured values of $n$ for natural sands appear in the literature.

Di Felice (1996) presents theoretical expressions for $w_s/w_t$ for viscous ($R_t < 0.2$) and inertial flow regimes ($R_t > 500$), again for spheres, but laboratory data are still required in the intermediate flow regime (corresponding to particle sizes in the range $0.07 < d < 2.5$ mm). Di Felice (1996) does not give values of $n$ for the viscous and inertial flow regimes, but appropriate values can be determined numerically from the given expressions that in fact vary slowly with voidage. These limiting theoretical values for $w_s/w_t$ lie just outside the experimental range determined by Richardson and Zaki (1954) and also in this study (see Section 4). Cheng (1997) proposed complex expressions for $n$ that were a function of both Reynolds number and volume concentration, which is not observed in most data, with the differences in settling velocity governed by changes in the viscosity of the fluid–particle mixture. Mandersloot et al. (1986) questioned the use of viscosity models and introduced a factor into Eq. (1) to account for particle flocing and the volume of fluid trapped close to the surface of rough particles. This factor needs evaluating via a complicated experimental procedure.

Consequently, given that there appeared to be no simple basis for estimating $n$ other than empirical correlations, at the outset of this work, we aimed to develop a straightforward method of determining appropriate values of $n$ which would apply to natural beach sands. This is verified against new laboratory data and values of $n$ extracted from published fluidisation data for marine sands (Wilhelm and Kwauk, 1948; Cleasby and Woods, 1975; Cleasby and Fan, 1981).
2.2. Predictive equations

Since Eq. (1) applies over the full range of possible voidage above incipient fluidisation, \( w_f/w_t \) ranges from 1 for \( e = 1 \) to some lower limit dependent on \( n \) for \( e_{\min} = 0.4 \) (\( c_{\max} = 0.6 \)), the approximate maximum concentration expected at the boundary with the undisturbed bed. Hence, to be consistent with the bed boundary condition at the point of fluidisation, when \( e = e_{\min} \):

\[
\frac{w_k}{w_t} \to \frac{w_f}{w_t} = (e_{\min})^n
\]

(2)

where \( w_f \) is the fluidisation velocity. This corresponds to the conditions when the upward seepage force just balances the immersed weight of the granular matrix above, usually written in soil mechanics terms as a critical hydraulic gradient, \( i \):

\[
i = \frac{1}{1 + e} = (s - 1)c_{\max}
\]

(3)

where \( s \) is the specific gravity of the granular material and \( e \) is the void ratio. The critical hydraulic gradient is typically in the range 0.8–1 for most quartz sediments, but higher for dense particles and lower for lightweight particles.

Hence, for any particular sediment, or cohesionless particle bed, the value for \( n \) which satisfies the bed boundary condition and clear water settling condition \( (w_k = w_t \) at \( c = 0 \), \( n \) arbitrary) may be approximated by:

\[
n = \frac{\log \left( \frac{w_f}{w_t} \right)}{\log (e_{\min})}
\]

(4)

The ratio \( w_f/w_t \) depends on grain density, grain size, \( e_{\min} \), fluid viscosity and shape and, for fine and medium sands, is typically of order 0.01–0.03. Baldock and Holmes (1998) discuss the implications of this small ratio when considering the effects of seepage flows on sediment transport over mobile beds. Of particular interest here is whether Eq. (4) also holds for naturally graded sands, in addition to uniform or well-sorted spherical particles, and this is investigated through the laboratory experiments described below. The fluidisation velocity is easily evaluated from appropriate expressions for the seepage flow in the bed and the bed permeability. In preparing this paper, we found that Eq. (4) was in fact derived previously by Godard and Richardson (1969), perhaps unsurprisingly given its simplicity. Richardson and Jeronimo (1979) used the same equation, as did Chianese et al. (1992), apparently independently; otherwise, Eq. (4) does not appear to have been used since and has not been tested for natural sands.

For small spherical particles settling at \( w_t \) according to Stokes Law, and with seepage flow expressed according to Darcy’s Law \( (w_f = k_i) \), then using Eqs. (3) and (4) and the simple expression for permeability (hydraulic conductivity) given by \( k = 10^{-3} gd^2/v \) (m/s) (Bear, 1972) gives:

\[
\frac{w_f}{w_t} = 10^{-3} c_{\max} 18 \approx 0.0108
\]

(5)

where \( g \) is gravity, \( d \) is the particle diameter, \( v \) is the kinematic viscosity (here of water) and \( c_{\max} = 0.6 \). Eqs. (4) and (5) give \( n = 4.94 \) for \( c_{\max} = 0.6 \), in fair agreement with the experimentally determined value for \( n \) of 4.8 for \( R_e < 0.2 \) (Khan and Richardson, 1989). Eq. (5) also gives very close agreement with the limiting (viscous regime) value of \( w_f/w_t = 0.011 \), which can be obtained from Di Felice (1995). However, in the same flow regime, the drag coefficient for sand is about 40% larger than that for smooth spheres (Fredsoe and Deigaard, 1992), and so \( w_f/w_t \approx 0.016 \), giving \( n = 4.5 \) for \( c_{\max} = 0.6 \) (i.e. \( e_{\min} = 0.4 \), the typical value for fine-medium sands).

For larger grain sizes, and denser particles, the value of \( n \) will reduce. In that case, Darcy’s Law may no longer be applicable and an appropriate expression for \( w_f \) may be obtained from a version of the Forchheimer equation (Ward, 1964; Bear, 1972)

\[
i = \frac{V}{(gk' + 0.55w_k)^{0.5}}\frac{0.55w_k}{\sqrt{gk'}}
\]

(6)

where \( k' \) is the intrinsic hydraulic conductivity, \( k' = kv/g \). Using the Kozeny–Carman equation (Bear, 1972) for the hydraulic conductivity in place of the simple formula above gives negligible differences when determining \( w_f \) or \( n \). Using Darcy’s Law, together with Eqs. (3) and (4), gives \( 3.67 < n < 4.45 \) for medium to fine sands \( (0.5 > d > 0.1 \text{ mm}; c_{\min} = 0.6) \). For \( d > 0.5 \text{ mm} \), \( w_f \) should be obtained from Eq. (6),
rather than from using Darcy’s Law (see Fig. 5 in Section 4). Hence, Eq. (4) quantifies the influence of grain size (and density) at high concentrations more precisely than taking a single value of \( n = 4 \) \((0.05 \, \text{mm} < d < 0.5 \, \text{mm})\) as suggested by Van Rijn (1984).

3. Experiments

In order to further investigate the hindered settling velocity described by Eq. (1) and to validate Eq. (4), experiments were carried out in a fluidisation column. The column has an internal diameter of 0.096 m and a height of 1.37 m (Fig. 1). An upward flow of water is generated through an inflow at the base, with a constant head maintained in the column via an overflow to a volumetric discharge measurement of \( w_s \). An equalising lower section of coarse gravel followed by a dense geotextile cloth filter and screen generates uniform flow across the column. A 0.1–0.15 m thick layer of sediment was allowed to settle through water from the top of the column onto the filter, preventing air entrainment and ensuring a normally compacted bed. Manometer tapping points at 0.05-m vertical intervals through the sidewall of the column enabled sampling of sediment at different elevations or measurement of the piezometric head within the sediment bed and sediment suspension.

Starting from a flow velocity less than \( w_t \), the flow rate was incrementally increased until fluidisation, and then expansion, of the bed occurred. For well-sorted sediments, the interface between the sediment suspension and the fluid above is very well defined, enabling an accurate assessment of the bed expansion relative to the initial bed level, and hence the concentration or voidage of the suspension. For graded sediments, the interface is not as sharp at high bed expansions and the concentration will vary within the column (Woodgate, 2002). However, the overall relative concentration can still be estimated to within about 5%. Experiments were carried out with a range of different particle sieve sizes, density and shape, with piezometric head differences and the bed expansion measured for flow rates up to \( w_f/w_t = 0.8 \). A summary of the particle properties is given in Table 1. The beach sands were naturally graded, with \( 1.6 < d_{50}/d_{10} < 2.7 \). The permeability, \( k \), terminal velocity, \( w_t \), and fluidisation velocity, \( w_f \), have been calculated as described in Section 2 \((c_{\text{max}} = 0.6)\), with drag coefficients for spheres or sand (Fredsoe and Deigaard, 1992) as appropriate. \( w_f \) was also determined experimentally, with fluidisation easily detected visually and through a rapid drop in the measured pressure gradient within the sediment bed. For calculation purposes, it has been assumed throughout the paper that \( c_{\text{max}} = 0.6 \), very close to the measured values obtained for the sands used in the experiments.

![Fig. 1. Fluidisation and settling column. At fluidisation, the volumetric flow velocity through the “stationary” bed is \( w_f \) and also corresponds to \( w_s \).](image-url)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Specific gravity, ( s )</th>
<th>( d (d_{50}) ) (mm)</th>
<th>( k ) (m/s)</th>
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<td>0.048</td>
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\(^a\) Intermediate axis length.
\(^b\) Measured value used for polystyrene prisms and anthracite.

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Grain size given is sieve diameter.
4. Results

Fig. 2 shows a plot of log $w_s$ versus log $e$ for the experiments described above. A log-linear relationship is apparent after fluidisation, with the slope of the line giving $n$, as illustrated by Richardson and Zaki (1954). The $y$-axis intercept at log($e_{\text{min}}$) varies for each sediment and is not a simple function of $w_t$ or particle diameter. Plotting $w_s/w_t$ against relative concentration, $c/c_{\text{max}}$, is perhaps more useful (Fig. 3), particularly if considering the influence of concentration on suspended sediment transport. At low concentrations ($0 < c/c_{\text{max}} < 0.2$), the settling velocity is reduced by up to 40%, with the reduction in settling velocity relatively insensitive to particle properties. In contrast, at high relative concentrations, the reduction in settling velocity is very dependent on the particle properties and ranges from 0.1$w_t$ to 0.02$w_t$ as $w_t$ decreases. Fig. 3 also shows the relative settling velocity in the viscous and inertial regimes obtained using values for $n$ as suggested by Richardson and Zaki (1954) and Di Felice’s (1996) expressions for $w_s/w_t$. Although both methods provide an upper and lower bound for $w_s/w_t$, typical values for natural sediments clearly fall in the intermediate regime and cannot be adequately described by the limiting values for $n$.

The curve for $n = 1$ is also shown, which represents the value of $n$ required to account for the upward displacement (or return flow) of fluid by falling particles (Fredsøe and Deigaard, 1992). A value of $n = 1$ was recently used in turbulent closure modelling of sediment transport by Li and Davies (2001) to include the effect of high concentrations on the settling velocity. Their model formulation required $c(1+n)<1$, which, for their near-bed reference concentration of $c = 0.3$, limited $n$ to the value 1. Such a model is therefore limited for application to sheet flow layers, but would be valid for suspensions up to about $c = 0.2$ (with $n < 4.5$ say). Suspended sediment concentrations of this magnitude typically occur in the swash zone (see Fig. 6 below). It is interesting to note that despite the fact that taking $n = 1$ clearly underestimates the hindered settling effect, Li and Davies (2001) found differences in predicted suspended concentrations of up to 30%. Using more realistic values of $n$ is therefore likely to lead to much greater differences in predicted near-bed sediment concentrations. Li and Davies (2001) additionally showed that turbulence damping at high sediment concentrations is also
important in controlling the overall sediment flux, and that both processes should be considered in modelling work.

Substituting Eq. (4) into Eq. (1) gives a new nondimensional equation which reduces the family of curves resulting from Eq. (1) and the experimental data to a single curve:

\[
\log\left(\frac{w_s}{w_f}\right) = \log(e) - \log(e_{\text{min}}) \tag{7}
\]

A plot of Eq. (7) for the present experimental results is given in Fig. 4a, with \(e_{\text{min}} = 0.4\). Here, in order to verify the accuracy of Eq. (7), \(w_f\) has been determined directly from the experimental data. The very close correlation between Eq. (7) and the experimental data suggests that the hydraulic properties of the granular matrix just prior to fluidisation provide an accurate measure of the suspension behaviour at much smaller concentrations, an intriguing relationship given the very different flow conditions. Eq. (7) holds over a wide range of concentrations (\(0.09 < c/c_{\text{max}} < 1\)), grain diameter and particle density. It also appears to apply to irregularly shaped particles (polystyrene, anthracite), provided the actual (measured) value for \(w_f\) is used. Fig. 4a suggests that Eq. (7) is accurate to within about \(\pm 10\%\) in terms of \(w_s\) and thus provides good estimates of the hindered settling effects. Alternatively, \(n\) can be calculated from Eq. (4), or read from Fig. 5 below, and then Eq. (1) can be used to determine \(w_s\).

A second plot of the same data, this time using the calculated values of \(w_f\), is shown in Fig. 4b. Calculating \(w_f\) from standard equations for permeability and flow in porous media makes little difference to the correlation between Eq. (7) and the data for the finer sediment (Fig. 4b). For the largest particles (\(d > 2\ mm\)), the correlation is not good at high concentrations because \(w_f\) is poorly predicted for these particles. At lower concentrations (\(c/c_{\text{max}} < 0.6\)), Eq. (7) again correlates well with the data. Hence, Eq. (4) appears to provide a simple but accurate measure of \(n\) for a wide range of flow conditions and particle properties.

Values for \(n\) obtained from Eq. (4) are presented graphically in Fig. 5 for sand particles with sieve diameters in the range \(0.06 > d_c > 1\ mm\). For grain diameters below 0.4 mm, the simpler Darcy’s flow law is sufficiently accurate. The values for \(n\) obtained directly from the slope of the curves in Fig. 2 are also
plotted for the present sand data (see Table 1). Additional values of $n$ have been extracted from similar fluidisation data for marine and filter sands presented by Wilhelm and Kwauk (1948), Cleasby and Woods (1975) and Cleasby and Fan (1981). The values of $n$ suggested by Richardson and Zaki (1954)
for uniform spheres are also shown. The measured values of $n$ for sand are significantly higher than estimates based on Richardson and Zaki (1954), but appear consistent with other fluidisation data for sand available in the literature. The good agreement between the data and the predicted values clearly indi-

Fig. 5. Predicted relationships between $n$ [Eq. (4)] and sieve grain size for sands, together with present experimental data and data extracted from cited sources. —: non-Darcy flow; ——: Darcy flow; ■: Richardson and Zaki (spherical particles); ■: natural beach sand (present data); ×: Wilhelm and Kwauk (1948) beach sand; ○: filter sand (present data); ○: Cleasby and Fan (1981) filter sand; +: Cleasby and Woods (1975) filter sand.

Fig. 6. Example of high relative suspended sediment concentration during a swash cycle. —: depth; ——: concentration; $c/c_{\text{max}}$.
cate that Eq. (4) provides a reliable method to determine appropriate values of $n$ for both sieved and naturally graded beach sands. In particular, using $d_{50}$ as a measure of the grain size in Eq. (4) appears appropriate for graded sands with $d_{90}/d_{10} < 3$. The difference between the values of $n$ for sand and those for equivalent-sized spheres can be significant. For example, for medium sands at a concentration of 0.4, the resulting settling velocity reduces to about 70% of the value predicted by Richardson and Zaki (1954). This difference appears to arise from the significant difference in drag coefficient between natural sand particles and spherical particles (Fredsøe and Deigaard, 1992).

The results above are simple to apply to typical coastal sediment transport problems. For example, Fig. 6 shows a typical example of an OBS record of the suspended sediment concentration (SSC) 1–2 cm above the bed in the swash zone, together with the associated swash depth. The data were recorded at Seven Mile Beach, NSW, Australia, a dissipative mildly sloping beach with $d_{50} = 0.15$ mm. The SSC during the uprush exceeds $0.2c_{\text{max}}$. Fig. 5 gives $n \approx 4.4$ ($d_s = 0.15$ mm), and then $w_s/w_t \approx 0.5$ is easily obtained from Eq. (1), i.e. a reduction in the sediment fall velocity of approximately 50%. Clearly, a change of this magnitude should be accounted for in future suspended sediment transport models for the swash zone.

5. Conclusions

At high sediment concentrations particle settling velocities reduce to some small fraction of their clear water settling velocity, leading to changes in near-bed sediment concentration gradients and a more uniform sediment distribution within the water column. The hindered settling velocity can be accurately predicted by the Richardson and Zaki (1954) equation, provided an appropriate value for the exponent $n$ is available for the particular material. A simple method of determining $n$ for sands has been illustrated and tested against new laboratory data from a fluidisation column. In particular, the hindered settling of natural sands has been investigated and appropriate values of $n$ are found to differ significantly from those suggested by Richardson and Zaki (1954) for spheres, with the hindered settling effect typically significantly greater for sand than for spheres of equivalent size. For fine and medium sands, the settling velocity reduces to less than 20% of the clear water settling velocity for suspended sediment concentrations greater than about 30%. Such concentrations are frequently reached, or exceeded, close to and within sheet flow layers during high sediment transport conditions.

Acknowledgements

The authors gratefully acknowledge funding from the CRC for Sustainable Tourism and University of Queensland staff grants. The positive and constructive review comments of Dr. A.G. Davies are much appreciated.

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